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TIE-33: Design strength of optical glass and ZERODUR®

1. Introduction

The design strength of a glass article is an essential characteristic when it has to survive mechanical or thermal loads during application. Frequently asked questions are:

- what is the strength of optical glass ?
- what is the strength of ZERODUR® ?
- what thickness is necessary for a ZERODUR® mirror blank with a specified diameter to endure the forces from attached actuators ?

Such questions concern the strength of glass directly or indirectly. In the following some general information will be given on the strength of glass. Additionally, a calculation scheme will be sketched which has been used for a variety of different applications. Data of common optical glass types are enclosed.

Recently it has been shown that a homogeneous isotropic load of 41 MPa (stress rate 2 MPa/s) on a 1 m^2 ZERODUR® surface (without any pre-damage), ground with bound diamond grain tool D151, leads to a failure probability of less than 0.1%, 95 % confidence level range. This stress level is 4 times higher than the conservative value of 10 MPa for a moderate surface treatment.

2. General aspects of glass strength

The strength of glasses and glass-ceramics is not primarily a material property like the Young's modulus e.g. The more prominent influence factors are:

- the crack depth distribution at surfaces and edges
- the size of the surface area exposed to tensile stress,
- the temporal behavior of the load and
- the environmental media.

All fracture of glass has at least two prerequisites: Tensile stress and (micro)flaws at the surfaces or the edges.

Measurement results for different surface conditions and further information on the bending strength of ZERODUR® can be found in chapter 6 and literature [4,5,6]. For low stress applications 10 MPa can be taken as a safe value for the strength of ZERODUR® as long as there are no heavily damaged surfaces or edges. Coarse grain worked surfaces (D151) show significantly higher strength values than 10 MPa. Surfaces where all micro cracks have been removed by etching lie even higher. For higher stress loads it is recommended to analyze the applicable strength limit in more detail referring to the procedures and data given in literature.

2.1 Microstructure condition of the glass surface

Milling and grinding of glass causes cracks at the surfaces. The depth of the surface cracks is roughly equal to the maximum grain size. As a rule, decreasing grains sizes result in less deep microcracks and in higher strength, and narrow grain size distributions lead to narrow strength distributions.

Polished surfaces have the highest strengths. However, this is valid only if the microcracks caused by preceding machining processes were eliminated. This can be achieved by subsequent grinding with decreasing grain sizes, each grinding process taking off a material layer at least three to four times as thick as the maximum crack depth from the preceding step. However, the strength distributions of glasses with polished surfaces scatter significantly more than those for glasses with ground surfaces.

2.2 Area of the tensile stress loaded surface

The admissible strength for a tensile stress loaded area decreases with increasing area. Deeper microcracks occur more frequently when the surface area increases.

2.3 Dependence on load history and environment

The fracture toughness of optical glass depends on the load history and the environmental surrounding. The higher the rate of stress increase is the higher is the bending strength of the glass part.

Environmental media influence the microcrack growth under long-term tensile stress loads. Liquid water enhances the crack growth and leads to lower strengths values consequently, whereas glass within vacuum exhibits the highest strength values.

Humid environment without stress loads are not detrimental to the strength of glass objects. On the contrary they are even beneficial to a certain extend.

2.4 Recommendations to maintain the glass strength

Cleaning procedures should use liquids in abundance and soft clothes. Avoid rubbing. Check the glass part for scratches. Scratches reduce the strength significantly!

Frames must not exert forces on glass parts. Avoid point contacts and direct metal contacts. Adhesive joints should use thick layers of soft glues, in order to compensate shear stresses.

3. Calculation procedure

3.1 Mathematical model

The commonly applied mathematical model, to fit the results of strength tests on glass parts~~,~~ is the Weibull distribution with two parameters.

$$F(\sigma) = 1 - \exp\left(-(\sigma / \sigma_0)^{\lambda}\right) \qquad (3\text{-}1)$$

with:

$F(\sigma)$ - Probability of failure at bending stress σ

σ_0 - Characteristic strength ($F(\sigma_0) = 63{,}21\ \%$)

λ - Weibull modulus (slope of the Weibull straight line and a measure for the scatter of the distribution.)

This distribution function is widely used in fracture statistics and allows deriving predictions on the failure rates for collectives of identical parts. Basing on laboratory test results obtained under well-defined conditions one can calculate design strengths for loads and conditions posed by special application requirements.

3.2 Outline of the calculation procedure

The calculation procedure described in the following is a simplified excerpt of the procedure that has been published in [1]. The notation used in this document is adopted from this publication.

From the evaluation of strength test results one obtains $\sigma_0(S_L, 63\%, R)$ and λ for a definite glass type and surface condition.

$\sigma_0(S_L, 63\%, R)$ is the characteristic strength for the laboratory test surface area S_L and stress increase rate R.

In the model employed the design strength depends on the parameters:

S_V - area of the tensile stress loaded surface

R_V - stress increase rate or

t_V - stress load duration time (when constant)

F_V - admissible probability of failure

The design strength is derived from the laboratory strength by dividing it by a factor $f_{FOS,}$ described in the following chapter.

$$\sigma_k = \sigma_0 / f_{FOS} \qquad (3\text{-}2)$$

3.3 Stress factors for safety of design

The factor f_{FOS} is factorized once again:

$$f_{FOS} = f_A \cdot f_P \cdot f_F \qquad (3\text{-}3)$$

The formulae for the individual factors are derived on the basis of the Weibull model using the laws on probability.

3.3.1 Area factor f_A

The area factor f_A is calculated according to the following formula

$$f_A = (S_V / S_L)^{1/\lambda} \qquad (3\text{-}4)$$

This formula assumes constant stress within the loaded area. It is a conservative approach since in many cases the tensile stresses have a maximum amount and fall off with increasing distances to that maximum. For a more rigid calculation one has to use $S_{eff,V}$ which is obtained by weighing the maximum stress with the stress distribution function instead of S_V.

3.3.2 Probability factor fp

The formula for the probability factor f_p is

$$f_p = \frac{1}{\left(\ln \frac{1}{1 - F_V}\right)^{1/\lambda}} \qquad (3\text{-}5)$$

3.3.3 Fatigue factor f_F

The general formula for the calculation of the fatigue factor f_F is

$$f_F = \left(\frac{t_{eff,V}}{t_{eff,L}}\right)^{1/n} \qquad (3\text{-}6)$$

where

$t_{eff,V}$ - effective loading time for the application

$t_{eff,L}$ - effective loading time at laboratory

n - environmental stress corrosion constant (for most optical glasses it is in the range from 12 to 20)

In the special case for a stress load constant in time this formula reads explicitly

$$f_F = (t_V \cdot f_A \cdot f_P \cdot R \cdot (n+1) / \sigma_0)^{1/n} \qquad (3\text{-}7)$$

For application cases with time varying loads $t_{eff,V}$ has to be calculated instead of t_V by using a weighing function that describes the load variation with time.

4. Bending strength of ZERODUR®

The following table and diagrams show measurement results for the characteristic strength, Weibull factor and stress corrosion constant n for ZERODUR®. The test procedure was according to DIN 52292-1 (ring-on-ring method) with a stress increasing rate of 2 MPa/s, room climate and a test area of 113 mm^2 (R30/6) (and 254 mm^2 for R45/9-experiments in 2008, marked by an asterisk in the tab.).

Material	**Surface condition**	**Characteristic strength σ_0 [MPa]**	**Weibull factor λ**	**Stress corrosion constant n / Medium**
ZERODUR®	SiC 600	108.0	16.0	51.7 Air 50% [2]
ZERODUR®	SiC 320	71.3	12.4	59.2 Air 50% [3]
ZERODUR®	SiC 230	57.5	15.7	30.7 Water [3]
ZERODUR®	SiC 100	53.6	18.7	
ZERODUR®	D 15 A	130.6	10.6	(for all
ZERODUR®	D 35	78.7	15.7	surfaces)
ZERODUR®	D 64	62.8*	11.0*	
ZERODUR®	D 151	54.1*	28.2*	
ZERODUR®	D 251	48.8	11.1	
ZERODUR®	Opt. polish	293.8	5.3	
ZERODUR®	D 64 etched	391.4*	3.0*	
ZERODUR®	D 151 etched	497*	3.4*	
ZERODUR®	SiC 600 at 77K	192.7	10.7	
glassy ZERODUR®	SiC 320	66.6	16.7	

Table 1: Characteristic strength, Weibull factor and stress corrosion factor of ZERODUR®
*= measurements from 2008 [5]. For more information on the surface condition data please refer to table 4 in the appendix.

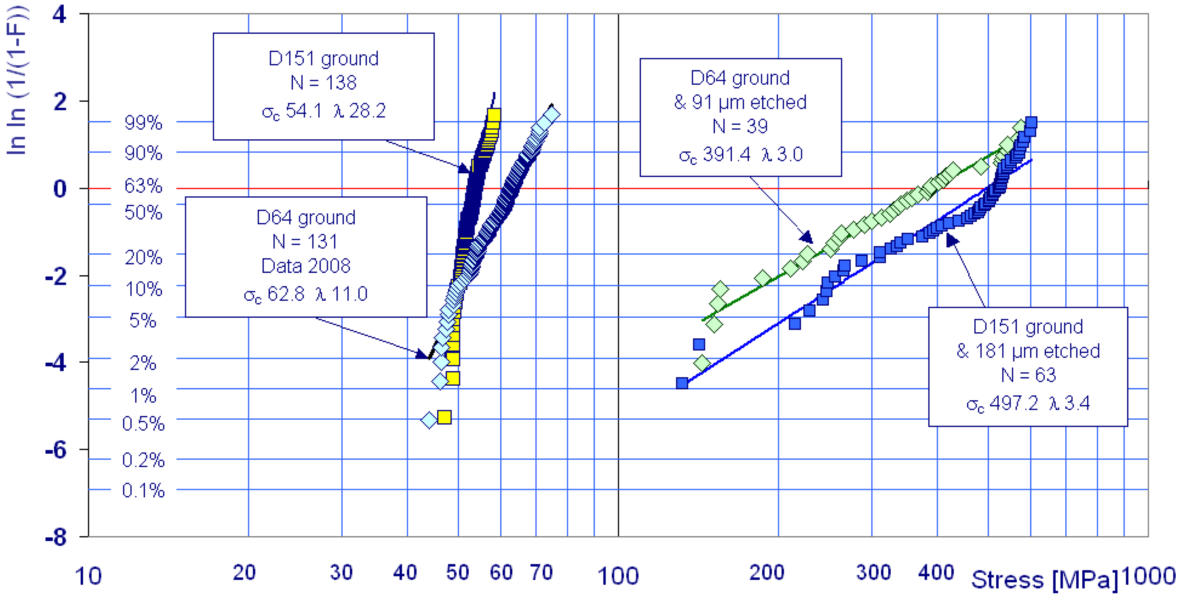

Figure 1: SCHOTT ZERODUR® D151 ground and etched samples together with D64 ground and etched samples [5].

Figure 1 shows the comparison of the fracture probability of samples with D64 and D151 ground surfaces. The D64 distribution has a significantly higher characteristic strength of 62.8 MPa than that of D151 with 54.1 MPa. However, the smaller slope of the D64 distribution leads to a crossover at the failure rate of 10 %. That means for lower target failure rates than 10 %, as will be requested in most application cases, for strength purposes it is not worthwhile to grind surfaces with D64 instead of D151.

Additional etching shifts the curves to higher strengths, as can be seen in the right part of figure 1. Results of D64 samples are shown, where 91 µm are removed by etching. Additionally, the result of D151 samples with a ground surface etched to 181 µm depth which is about 50 % higher than the maximum micro crack depths for D151 surfaces of about 120 µm. The D151 etched distribution (characteristic strength: 497 MPa / slope: 3.4) lies higher than that of D64 ground and etched surfaces. Obviously the grain size of the preceding grinding process does not play any decisive role anymore, when the etched off layer thickness is significantly higher than the subsurface crack depths. This result is also of practical significance. It means that it is not necessary to prepare surfaces with fine grinding which are to be etched later on as long as a minimum layer thickness is etched off, which is higher than the maximum crack depth.

It is noticeable, that both distributions for the samples with the ratio of 1.5 between the etch depth have similar slopes. On the other hand, they have significantly different characteristic strengths.

The results of the D151 measurement can be extrapolated to application conditions. The general procedure and methods are described in [4] and [6]. In [4] it was shown that a homogeneous isotropic load of 41 MPa (stress rate 2 MPa/s) on a 1 m^2 ZERODUR® surface (without any pre-damage), ground with bound diamond grain tool D151, leads to a failure probability of less than 0.1%, 95 % confidence level range. This stress level is 4 times higher than the conservative value of 10 MPa for a moderate surface treatment.

It can be generally stated that etching or optical polishing of ground surfaces significantly improves the characteristic strength of the glass. Etching broadens the strength probability distribution thus leading to a smaller slope in the cumulative failure probability plot. These statements are only valid if the surfaces are not damaged by handling etc.

5. Bending strength of optical glass

The following table shows some measurement results for the characteristic strength, Weibull factor and stress corrosion constant n for selected optical glass types. The test procedure was according to DIN 52292-1 Double ring method R 30-6 with a stress increasing rate of 2 MPa/s, room climate and a test area of 113 mm^2.

Material	**Surface condition**	**Characteristic strength σ_0 [MPa]**	**Weibull factor λ**
N-BK 7	SiC 600	70.6	30.4
N-BK 7	D 64	50.3	13.3
N-BK 7	D 64 etched	234.7	4.1
N-ZK 7	SiC 600	68.9	14.1
N-BaK 1	SiC 600	58.9	8.2
N-SK 16	SiC 600	62.3	19.3
N-LaK 8	SiC 600	70.0	29.9
F 2	SiC 600	57.1	25.0
N-BaSF 64	SiC 600	70.1	23.9
N-LaF 21	SiC 600	75.9	28.6
SF5	SiC 600	55.3	10.6
SF6	SiC 600	49.2	5.4
SF 57	SiC 600	39.1	14.7
KzFS N4	SiC 600	49.4	25.5
UG 11	SiC 600	63.4	4.4
KG 3	SiC 600	61.4	11.3

Table 2: Characteristic strength and Weibull factor of optical glass.

6. Fracture toughness of glass

So far the strength of optical glass was considered from a statistical point of view by measuring the bending strength of samples and subsequently estimating the failure probabilities. Looking at a single flaw in a material the maximum bending strength depends on the size of the flaw and geometry in the material. For example, for of a flaw with a short depth in a thick plate with tensile forces acting normal to the crack plane (so called “Mode I” or “tensile mode” or “crack opening mode”) one can define a stress intensity factor K_I by

$$K_I = Y\sigma_0\sqrt{a} \approx 2\sigma_0\sqrt{a} \qquad (6\text{-}1)$$

with σ_0 being the nominal stress perpendicular to the crack plane and a the depth of the flaw. Y is a geometry related factor. A flaw will result in a fracture if K_I reaches a critical value.

$$K_I \geq K_{IC} \qquad (6\text{-}2)$$

K_{IC} is the fracture toughness for fracture mode I (tensile forces normal to the crack plane, crack propagation perpendicular to the forces). K_{IC} is a material constant. For glasses without additional strengthening the value is typically ≤ 1. Table 3 gives fracture toughness values of some glasses:

Glass	**K_{IC} [MPa $m^{1/2}$]**
N-BK7	1.1
F5	0.9
ZERODUR®	0.9
SF6	0.7

Table 3: Fracture toughness values of some glasses [9,10]

In principle, therefore, if one knows the crack geometries of all of the cracks in a part and the fracture toughness of the material, then one can predict the strength of that part. As a practical matter, it is not yet possible to know the crack geometries of all of the cracks in a part exactly enough in order to make reliable predictions of the strength of the part [11].

For a given nominal stress the plate will break for a critical crack depth a_c of

$$a_c \approx \left(\frac{K_{IC}}{2\sigma_0}\right)^2 \qquad (6\text{-}3)$$

Numerical example: For the characteristic strength of ZERODUR® of samples with D64 surface condition $\sigma_0 \approx 64$ MPa (table 1) and ZERODUR® $K_{IC} \approx 0.9$ MPa $m^{1/2}$ the critical flaw size a_c is approx. 49 µm. This flaw size compares to the grain sizes for D64 bonded diamond grains (table 4).

All oxidic glasses and ceramics subjected to tensile stress at their surfaces in a humid environment exhibit a slow crack growth as long as the stress intensity factor lies well below the critical value. The velocity of the crack growth can be approximated by [12]:

$$\frac{da}{dt} = A * \left(\frac{K_I(a)}{K_{IC}} \right)^n \qquad (6\text{-}4)$$

a denotes the depth of the crack, A is a material specific constant and n is the stress corrosion constant depending on the material and environmental humidity. As a rule, the drier the environment is, the higher is n. For glass and glass ceramic materials it is known that in humid environments without stress load the strength gets even higher to a certain extent, due to a healing effect (rounding of the microcrack tips due to chemical corrosion).

Typical values for the stress corrosion constant can be found in table 1 and 2. Typically the sub critical crack grow can start from 0,25*K_{IC}.

7. Appendix: Grain Sizes

	Designation	Mean size [µm]	Max. size [µm]	ASTM equiv.
Bonded diamond grains **acc. DIN 848**	D251	231	250	60/70
	D151	138	150	100/120
	D107	98	106	140/170
	D64	58	63	230/270
	D35	36	40	--
	D15A	12.5	15	--
Loose silicon-carbide grains **acc. FEPA Std. 42-D-1984**	SiC 100	116	149	--
	SiC 230	53	84	--
	SiC 320	29	49	--
	SiC 600	9	19	--

Table 4: Grain sizes of grinding tools

9. Literature

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